NAME: ____________________

YEAR 2 | FUNCTIONS

VIDEO PLAYLIST LINK: parkermaths.com/link/functionsplaylist

TYPES OF NUMBER | KEY FACTS

- *Natural numbers* $(\mathbb{N})$ are *positive whole numbers*.
- *Integers* $(\mathbb{Z})$ are *positive* and *negative* whole numbers, including *zero*.
- *Rational numbers* $(\mathbb{Q})$ are numbers that can be *expressed as fractions*.
- *Real numbers* $(\mathbb{R})$ extend the rational numbers to include *irrational numbers* (e.g. $\sqrt{2}\,,\ \pi\,, e$)

MAPPINGS AND FUNCTIONS | KEY FACTS

- A *mapping* defines a relationship between a set of input values and output values.
- A *function* is a mapping where *each input maps to a single output*.

DOMAIN AND RANGE | KEY FACTS

- The *domain* of a function is the *set of input values* for which it is defined.
- The *range* of a function is the *set of possible output values*.

FUNCTION NOTATION | KEY FACTS

The following types of function notation are used in the A-Level

THE DOMAIN

THE RULE

$\mathrm{f}(x) = 3x + 2 \qquad -6 < x \le 7\ ,\ x \in \mathbb{R}$

$\mathrm{f} : x \mapsto 3x + 2 \qquad x \in (-6, 7]\ ,\ x \in \mathbb{R}$

x IS A REAL NUMBER

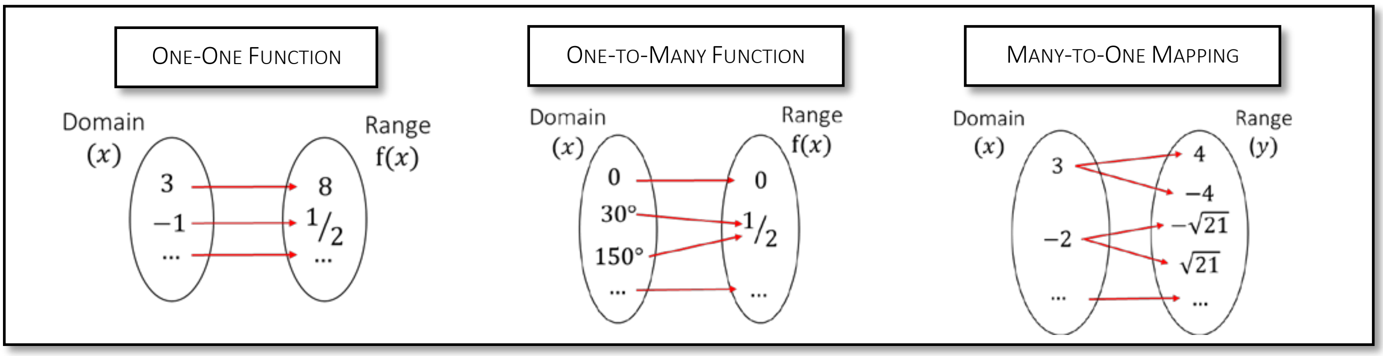
THE CURVED BRACKET IMPLIES -6 IS NOT INCLUDED IN THE DOMAIN

THE SQUARE BRACKET IMPLIES 7 IS INCLUDED IN THE DOMAIN

TYPES OF MAPPING | KEY FACTS

- One-to-one function: Each input value generates a *unique output value*.
- Many-to-one function: Two or more input values can generate the same output value.
- One-to-many mapping: *Each input value* can generate more than one output.
    - o *One-to-Many* mappings are *not functions*.

A ONE-TO-ONE FUNCTION HAS NO TURNING POINTS IN THE SPECIFIED DOMAIN.



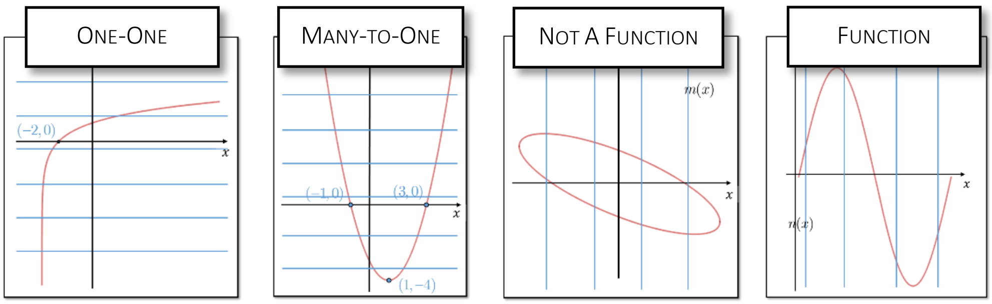
HORIZONTAL AND VERTICAL LINE TESTS | KEY FACTS

HORIZONTAL LINE TEST:

- Draw a set of horizontal lines
- If any line crosses the curve more than once, it is *not* a one-one function.

VERTICAL LINE TEST:

- Draw a set of vertical lines
- If any line crosses the curve more than once, it is *not* a function.



DOMAIN AND RANGE | EXAMPLE 1

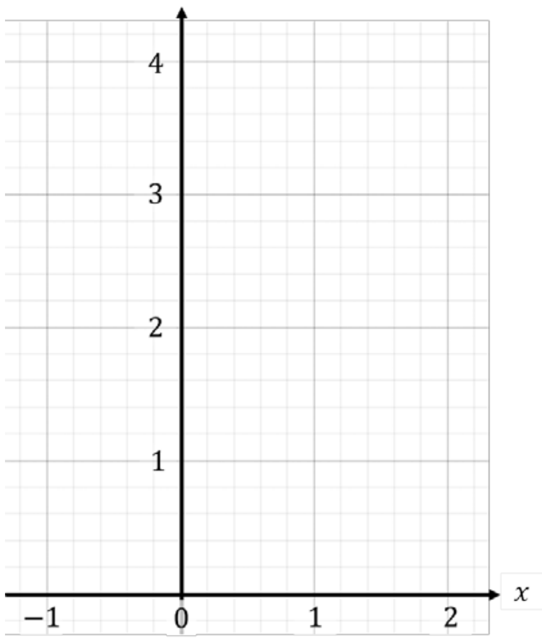
A function f is defined such that $\mathrm{f}(x) = x^2 \quad -1 < x \leq 2\ ,\ x \in \mathbb{R}$.

(a) Sketch the graph of $y = \mathrm{f}(x)$ on the axis provided.

(b) State the range of the function.

(c) Evaluate $\mathrm{f}(1.5)$.

(d) Find the exact solution to the equation $\mathrm{f}(x) = 3$.



DOMAIN AND RANGE | PROBLEM 1

A function g is defined such that $\mathrm{g}(x) = \sin x \quad 0 \leq x \leq \dfrac{3\pi}{2}\ ,\ x \in \mathbb{R}$

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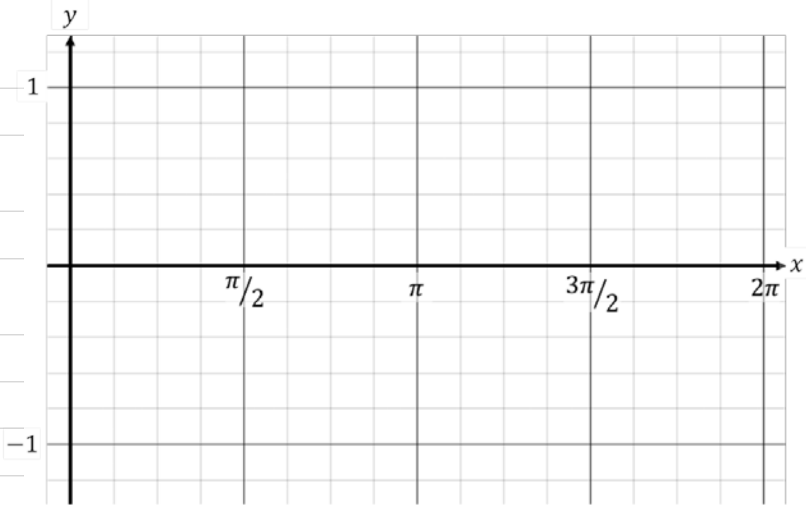
(a) Sketch the graph of $y = \mathrm{g}(x)$ on the axis provided.

(b) State the range of the function.

(c) Evaluate $\mathrm{g}(1)$.

(d) Find the exact solution to the equation $\mathrm{g}(x) = -0.5$.

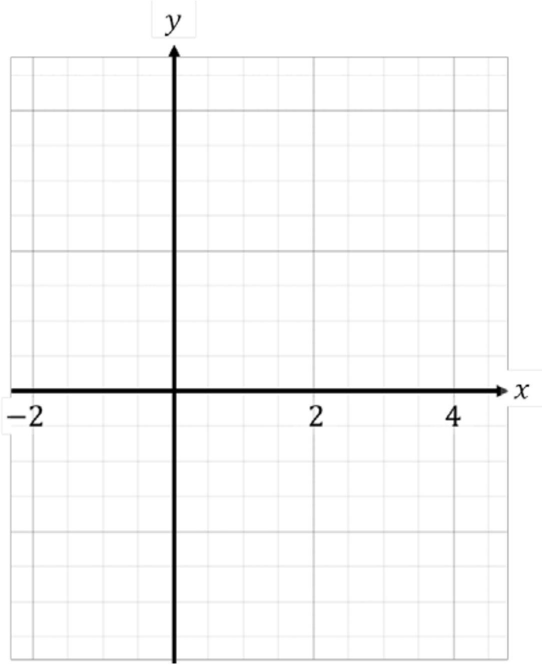


DOMAIN AND RANGE | EXAMPLE 2

(a) Find the greatest possible domain for the function $\mathrm{h} : x \mapsto \dfrac{1}{x-2}$

(b) State the range of $\mathrm{h}(x)$.

(c) Sketch the graph of $y = \mathrm{k}(x)$ on the axis provided.

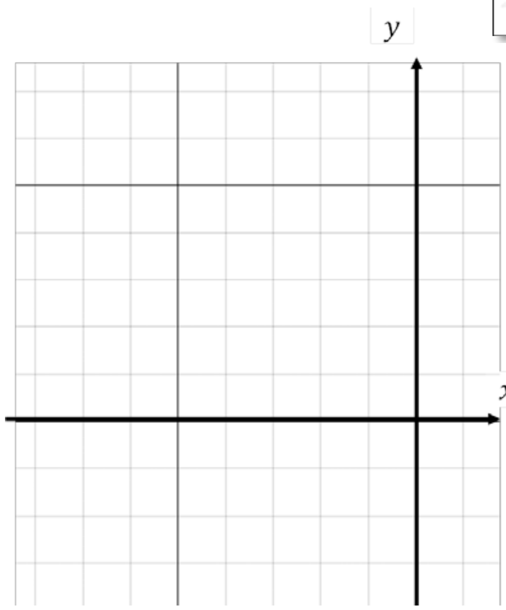


DOMAIN AND RANGE | PROBLEM 2

(a) Find the greatest possible domain for the function $\mathrm{k} : x \mapsto \dfrac{1}{x+5} + 2$

(b) State the range of $\mathrm{k}(x)$.

Sketch the graph of $y = \mathrm{k}(x)$ on the axis provided.

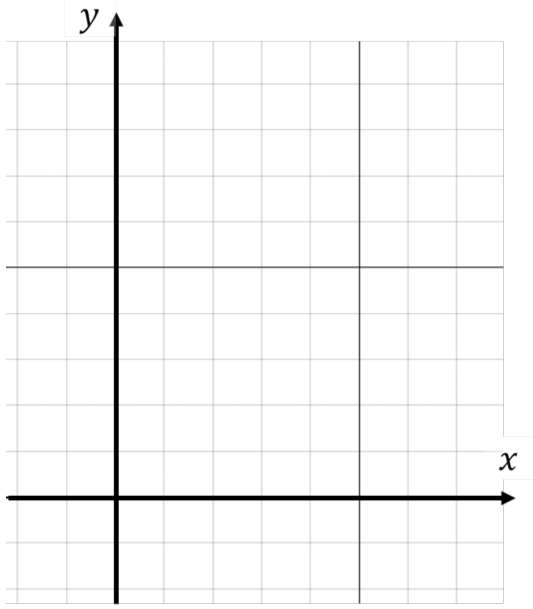


DOMAIN AND RANGE | EXAMPLE 3

A function g is defined such that $\mathrm{g}(x) = x^2 - 6x + 8$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = \mathrm{g}(x)$, indicating all intercepts and stationary points.

(b) State the range of g .

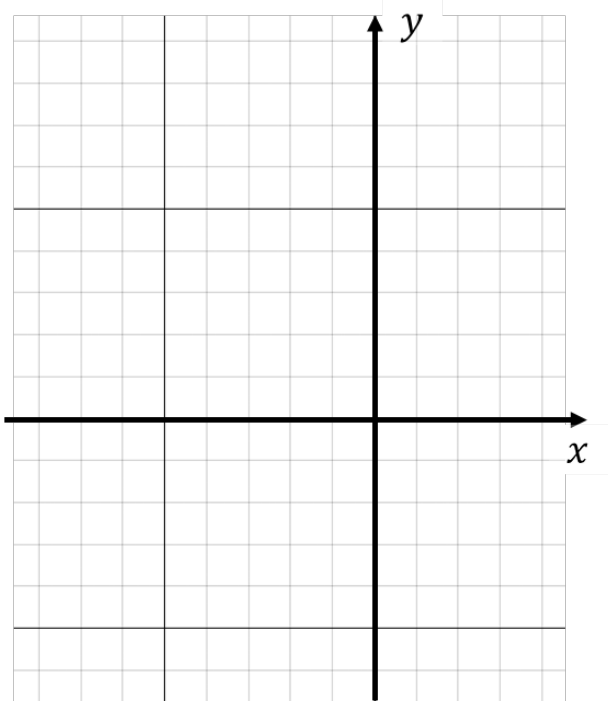


DOMAIN AND RANGE | PROBLEM 3

A function h is defined such that $\mathrm{h}(x) = -x^2 - 4x + 5$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = \mathrm{h}(x)$, indicating all intercepts and stationary points.

(b) State the range of h .



COMPOSITE FUNCTIONS | EXAMPLE-PROBLEM PAIRS

The functions f, g and h are defined as follows:

$$\mathrm{f}(x) = 2x - 3 \ , \ x \in \mathbb{R} \qquad , \qquad \mathrm{g}(x) = 4x^2 \ , \ x \in \mathbb{R} \qquad , \qquad \mathrm{h}(x) = {}^{1}\!/_{x} \ , \ x \neq 0 \ , \ x \in \mathbb{R}$$

(a) Find $\mathrm{fg}(x)$.

(b) Find $\mathrm{gf}(x)$.

USUALLY, $\mathrm{fg}(x) \neq \mathrm{gf}(x)$

(c) Find $\mathrm{hfg}(2)$.

(d) Find $\mathrm{fgh}(3)$.

MOST OF THE TIME IT IS EASIER TO SUB IN THE VALUE FIRST

(e) Find $\mathrm{f}^2(4)$.

(f) Find $\mathrm{g}^2\mathrm{h}(2)$.

$\mathrm{f}^2(x) \equiv \mathrm{ff}(x)$

INVERSE FUNCTIONS | KEY FACTS

- Only one-one functions have an inverse.
- A function and its inverse are related as follows:

Domain of $\mathrm{f} \Leftrightarrow$ Range of f^{-1} Range of $\mathrm{f} \Leftrightarrow$ Domain of f^{-1} The graph of $\mathrm{f}^{-1}(x)$ reflects the graph $\mathrm{f}(x)$ in the line $y = x$

INVERSE FUNCTIONS | EXAMPLE PROBLEM PAIR 1

A function f is defined by $\mathrm{f}(x) = 2e^{5x} + 7 \ , \ x \in \mathbb{R}$.

Find the inverse function f^{-1}.

A function g is defined by $\mathrm{g}(x) = \sqrt{2x^2 - 5} \ , \ x \geq \sqrt{2.5} \ , \ x \in \mathbb{R}$.

(a) Find the inverse function g^{-1}.

INVERSE FUNCTIONS | EXAMPLE PROBLEM PAIR 2

Given that $\mathrm{h}(x) = \dfrac{2x+1}{x-3}$, $x \neq 3$, $x \in \mathbb{R}$.

Find the inverse function h^{-1} .

A function g is defined by $k(x) = \dfrac{2-5x}{3x+1}$, $x \neq -\dfrac{1}{3}$, $x \in \mathbb{R}$.

Find the inverse function k^{-1} .

INVERSE FUNCTIONS | EXAMPLE PROBLEM PAIR 3

Given that $\mathrm{g}(x) = x^2 + 2x + 3$, $x \geq -1$, $x \in \mathbb{R}$.

(a) By completing the square, find the range of $\mathrm{g}(x)$

(b) State the greatest possible domain of $\mathrm{g}^{-1}(x)$.

(c) State the range of $\mathrm{g}^{-1}(x)$.

(d) Find the inverse function $\mathrm{g}^{-1}(x)$.

Given that $\mathrm{h}(x) = 2x^2 + 8x - 5$, $x \leq -2$, $x \in \mathbb{R}$.

(a) By completing the square, find the range of $\mathrm{h}(x)$

(b) State the greatest possible domain of $\mathrm{h}^{-1}(x)$.

(c) State the range of $\mathrm{h}^{-1}(x)$.

(d) Find the inverse function $\mathrm{h}^{-1}(x)$.

TO FIND THE INVERSE OF A QUADRATIC FUNCTION, COMPLETE THE SQUARE